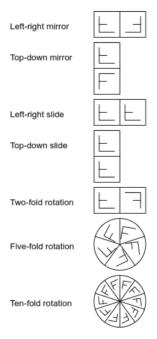
Girih

Girih is a 10th century art form invented by the Persians (now Iran). They discovered a set of geometric polygon tiles that would tessellate, that is, cover a plane without gaps. Some patterns have left-right and top-down symmetries, so can be used as wall papers. Many other patters are radially symmetric, meaning that the pattern can be revolved around the center with no difference in pattern. Some radially symmetric patterns are 2-fold, in that the pattern is the same with a 180 degree rotation. Other patterns are 5-fold symmetric meaning that the pattern must be rotated 72 degrees to be the same. There are even some 10-fold symmetric patterns that are the same after a rotation of 36 degrees. Girih tiles can tessellate with no repeating pattern at all in a so-call asymmetric design.

The Persians included at least one other characteristic to the tiles. They added strapping that forms knots when the tiles are laid. Indeed "girih" means knot in Persian. The Persians would use the tiles to form complex knot designs and then obscure that they used tiles. The tiles could be manufactured using a mold, so every tile would be like every other tile. In this way, an architect could specify a complex knot design and have it executed by relatively untrained artisans. Some tiles had a second level of knot design for even more complex designs.

Symmetry

Before the basic tiles are introduced, it would be good to have a little discussion about symmetry. Symmetry is a property of a object where part of the object is like another part of the object. Symmetry comes in many flavors.



decagon-loop-with-hex-bow-tie

Simple symmetry is just a step and repeat pattern, almost as if the pattern were stamped over and over. This type of symmetry is called a "slide." Usually this is a side-by-side slide, or left-right slide. It can also be a vertical repeat or a top-down slide. It also can be both simultaneously.

Another type of symmetry is mirroring, where a side of a pattern is a mirror image of the other side. Again mirroring is typically rightleft, top-bottom or both.

The pattern can be rotated when repeated. This is similar to a slide except that the pattern is rotated when repeated.

If a rotating pattern revolved around a single point, then the pattern is said to have a number of folds. The girih patterns may have 2-fold, 5-fold and 10-fold rotational symmetries. As the name suggests a two-fold pattern has two copies of a patters revolved or folded about a point. The five and ten fold patterns are similar, but have the pattern repeats of 1/5 or 1/10 of the overall pattern.

The girih tiles use polygons with angles that are multiples of 36 degrees. This means that any girih pattern will only use lines that are 36 degrees apart. So a girih design appears to use only 5 directions. Lines that are 180s apart appear to be in the same direction.

The girih strapping is place so that it is 54 degrees from the face of the polygon. 54 degrees is 3 times 18 degrees, so the girih strapping appears to use only 5 line directions, although the strapping lines are 18 degrees (or half way) off of the basic girih tiles lines.

Girih tiles use only five equilateral polygons: a decagon (10 sided regular polygon), pentagon (5-sided regular polygon), a rhombus (4-sided polygon), a hexagon (a 6-sided polygon elongated to use angles of multiples of 36 degrees) and a bow-tie (a 6-sided polygon that is concave (bends inward). While these are complete, additional interesting patterns can be formed using a lozenge or narrow rhombus.

Basic Girih Tiles

The ancient girih has only five tile shapes: decagon, pentagon, elongated hexagon, bow-tie and rhombus. A sixth shape, the lozenge, is added here because it is interesting /and allows many more patterns. The lozenge is similar to the rhombus, except that its narrow angle is half as big. Sir Roger Penrose, who researched several tessellations including one of this shape. This polygon was described by Albrecht Durer in 1525.

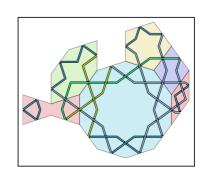
The following table discusses the various characteristics and differences of each tile shape.

Name	Figure	Number of Sides	Rotation Symmetry	Mirror Symmetry	Regular Angles	Tessellate on its Own	Opposite Sides Parallel
decagon		10	2-, 5-, 10-fold	through vertices or midpoints	yes, 36°	no, simplest tessellation is with bow- ties	yes
hexagon		6	2-fold	through long or short axis	no, 36°, 36°, 108°	yes	yes
bow-tie		6	2-fold	through long or short axis	no, -18°, 108°, 108°	yes	yes
pentagon		5	5-fold	through a vertex or midpoint	yes, 72°	no, simplest tessellation is with lozenges	no
rhombus		4	2-fold	long or short axis	no, 72°, 108°	yes	yes
Lozenge*		4	2-fold	through long or short axis	no, 36°, 144°	yes	yes

• Five Plus One Shapes Hooked Up

This figure shows the five girih tiles plus the lozenge tile with their edges touching.

Source: src/5+1-girih-tiles.json



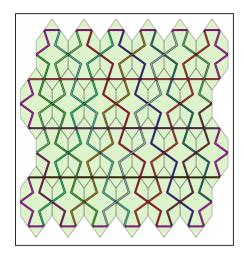
This section begins to build girih tessellations with simple symmetry: 2-fold rotational symmetry, right-left mirror symmetry or top-bottom mirror symmetry. Some patterns have all three.

• 2-fold Hexagon Tessellation

This shows a simple tessellation using only the girih hexagon tile.

This pattern can also have right-left mirror symmetry and top-bottom mirror symmetry (for odd number of rows). The pattern also can slide horizontally or vertically.

Source: src/2-fold-hexagon.json

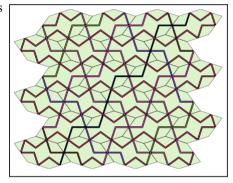


• 2-fold Hexagon Herringbone Tessellation

This shows a tessellation using only the girih hexagon tile. Here each row of tiles uses an alternating angle to form a herringbone pattern.

This pattern does not right-left mirror symmetry or top-bottom mirror symmetry, although the pattern can slide horizontally or vertically.

Source: src/2-fold-hexagon.json

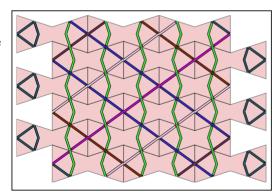


• 2-fold Bow-Tie Tessellation

This is a simple tessellation using only bow-tie polygons.

This pattern can have left-right and top-down symmetry. It also can slide vertically or horizontally.

Source: src/2-fold-bow-tie.json

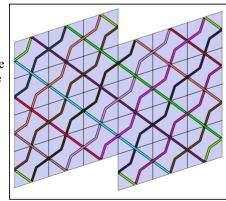


• 2-fold Rhombus Tessellation

This is a simple tessellation using only rhombus polygons.

This pattern can have left-right and top-down symmetry if the major and minor axes are aligned vertically and horizontally. The pattern can slide vertically and can almost slide horizontally.

Source: src/2-fold-rhombus.json

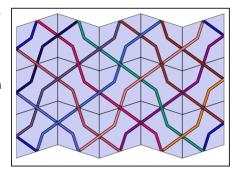


• 2-fold Rhombus Herringbone Tessellation

This is a tessellation using only rhombus polygons, but the angle of the rhombus is alternating in every row to form a herringbone pattern.

This pattern can have left-right symmetry when even pairs of rhombi are used horizontally. Top-down symmetry requires a shift of one rhombus. As long as pairs of rhombuses are considered, the pattern can slide horizontally. The pattern can slide vertically.

Source: src/2-fold-rhombus-herringbone.json

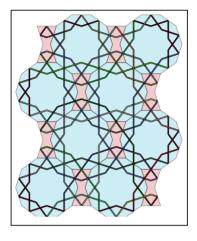


• 2-fold Bow-Tie and Decagon Tessellation

This is a tessellation using the decagon and bow-tie polygons.

This pattern can have left-right and top-down symmetry if the major and minor axes of the bow-ties are aligned vertically and horizontally. This pattern can slide vertically or horizontally.

Source: src/2-fold-decagon-bow-tie.json

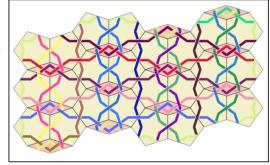


• 2-fold Pentagon and Lozenge Tessellation

This tessellation uses the pentagon and lozenge tiles. Since this tessellation uses the lozenge tiles, so it is not a true girih tiling.

This pattern can have left-right and top-down symmetry. This pattern can slide vertically or horizontally.

Source: src/2-fold-pentagon-lozenge.json

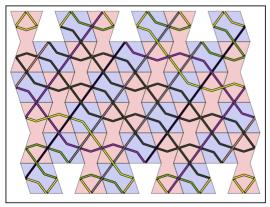


• 2-fold Bow-Tie and Rhombus Tessellation

This pattern has two-fold rotational symmetry.

This pattern can not have left-right and top-down symmetry, although the pattern can slide horizontally or vertically.

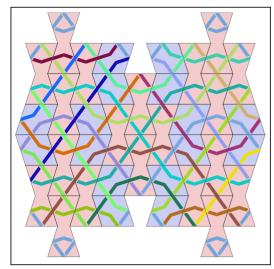
Source: src/2-fold-bow-tie-rhombus.json



• 2-fold Bow-Tie and Rhombus Alternate Tessellation

This pattern has is slightly different than the previous pattern. It has two-fold rotational symmetry. This pattern can have left-right and top-down symmetry, and the pattern can slide horizontally or vertically.

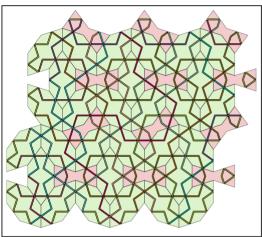
Source: src/2-fold-bow-tie-rhombus.json



• 2-fold Hexagon and Bow-Tie Tessellation

This pattern has a larger unit cell than most because of the "twisting" of the bow-tie tiles. It has two-fold rotational symmetry. This pattern can have left-right and top-down symmetry, and the pattern can slide horizontally or vertically.

Source: src/2-fold-hexagon-bow-tie.json



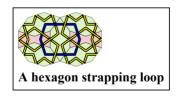
Strap Loops

The straps can make interesting patterns themselves. To make the special strapping patterns repeat, the underlying tile pattern must be repeated. The following are samples of possible strap loops

Hexagon Strapping Loop

The strapping forms a hexagonal loop.

Source: src/hexagon-loop.json



• Pentagon Strapping Loop

The strapping forms a pentagonal loop. Note that some of the corners of the pentagon loop also form smaller pentagons. There are several tile combinations to form this pattern.

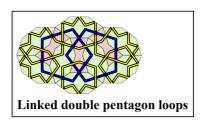
Source: src/pentagon-loop.json



• Double Pentagon Strapping Loops

The strapping forms two pentagonal loops that are intertwined or linked.

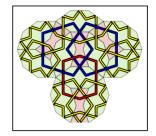
Source: src/double-pentagon-loop.json



• Double Pentagon and Hexagon Strapping Loops

The strapping forms two pentagonal loops that are intertwined or linked. Both pentagons are linked by a hexagonal loop.

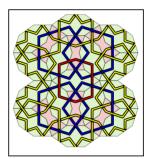
Source: src/hexagon-loop.json



• Double Double Pentagon and Hexagon Loops

The strapping forms a pair of two pentagonal loops that are intertwined or linked. All four pentagons are linked by a hexagonal loop.

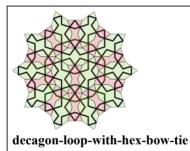
Source: src/double-double-pentagon-hexagon-loop.json



• Decagon Strapping Loop

The strapping in this figure forms a decagon. The center hexagons and bow-tie tiles can be replaced with decagon tile.

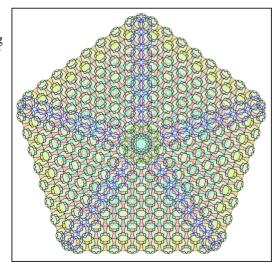
Source: src/decagon-loop-with-hex-bow-tie.json



• Mostly Hexagon Strapping Loops

The strapping in this figure forms a central decagon, surrounded by ten looping pentagons. Mostly there are hexagonal loop. On the spines there are a series of linked pentagon loops. The edges are crowns.

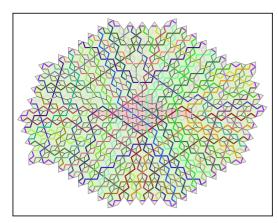
Source: src/5-fold-hexagon-loops.json



• 2-fold Strap Loops

This figure shows a simple pentagon composed of decagons and bow-ties.

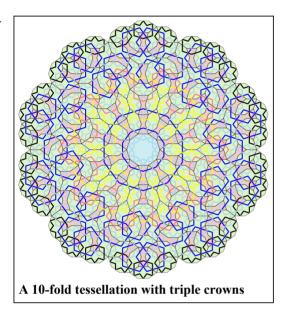
Source: 2-fold-bh.json



• 10-fold Tessellation with Double Pentagons and Triple Crown Edges

This design build a 10-fold pattern that has triple crown strapping edges. The triple crown is the same thing as a linked double pentagon loops.

Source: src/10-fold-dbh-triple-crown.json



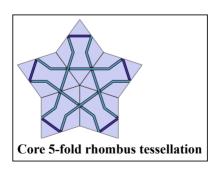
5-fold Tessellations

5-fold tessellations have rotational symmetry. By definition they cannot have sliding or mirror symmetries, however the can within each sector. This provides an interesting way to have lots of symmetry that goes beyond the simple rotational symmetry.

• 5-fold Rhombus Tessellation

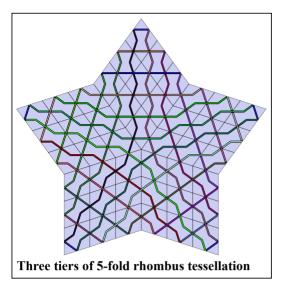
This is five-fold tessellation made with only rhombus polygons. This type of tessellation is useful in creating other radially symmetric patterns by creating subpatterns that are rhombus, and then repeating the sub-pattern.

Source: src/5-fold-rhombus-core.json



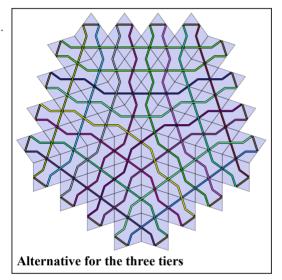
The second figure shows filling out three tiers of the tessellation radially and keeping the general star shape.

Source: src/5-fold-rhombus-tiered.json



The third figure shows filling out three tiers of the tessellation radially but filling the valley between star points to form a saw-toothed pentagon.

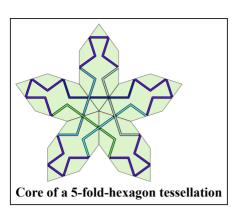
Source: src/5-fold-rhombus-tiered2.json



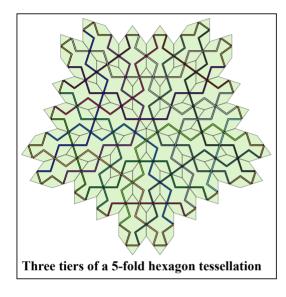
• 5-fold Hexagon Tessellation

This 5-fold tessellation is made with hexagon tiles. It has similarities with the preceding tessellation using rhombus tiles.

Source: src/5-fold-hexagon-core.json



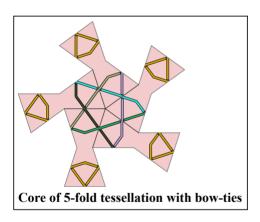
Source: src/5-fold-hexagon-tiered.json



• 5-fold Bow-Tie Tessellation

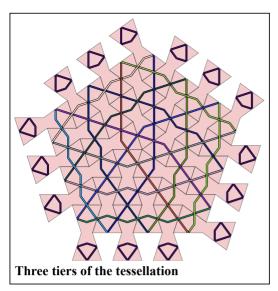
This five-fold tessellation is mad using only bow-tie tiles.

Source: src/5-fold-bow-tie-core.json



This shows the tessellation expanded through three tiers.

Source: src/5-fold-bow-tie-tiered.json

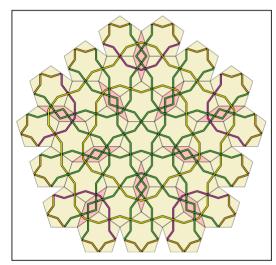


• 5-fold Pentagon and Lozenge Tessellation

A regular pentagon which will not tessellate. It is made to tessellate here with the addition of a lozenge, originally described for this purpose by Albrecht Durer in 1525.

The tiling was used in the Pilgrimage Church of Saint John of Nepomuk at Zelena Hora, Czech republic, built starting in 1719. [Photo from commons.wikimedia.org]

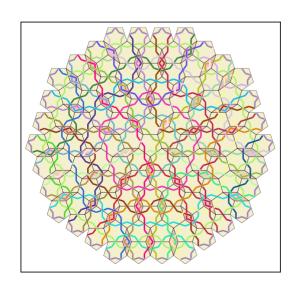




Source: src/5-fold-pentagon-lozenge.json

Same patterns as previous with more tiers.

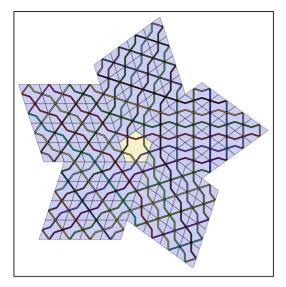
Source: src/5-fold-pl.json



• 5-fold Pentagon and Rhombus Tessellation

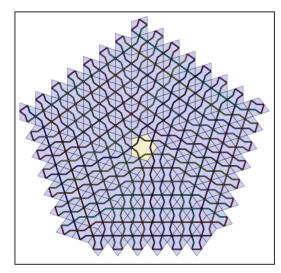
A regular pentagon will not tessellate. The rhombus will tessellate, here in each sector. This is similar to the simpler 5-fold rhombus tessellation, in that here each sector is offset by the central pentagon.

Source: src/5-fold-pentagon-rhombus.json



This is the same base tessellation as the previous figure, although here the valleys between each rhombus is filled in to from a jagged pentagon shape rather than a pinwheel or star shape.

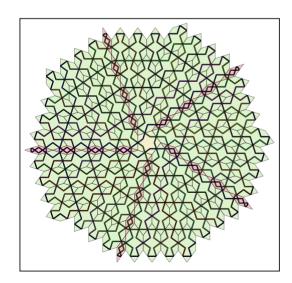
Source: src/5-fold-pentagon-rhombus-filled.json



• 5-fold Pentagon, Hexagon and Lozenge Tessellation

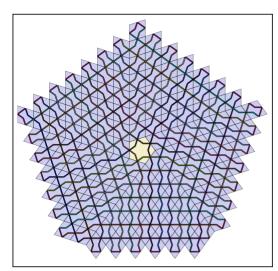
This tessellation has a central pentagon, built out with hexagons. The sectors are separated by a string of lozenge tile.

Source: src/5-fold-pentagon-hexagon-lozenge.json



This is the same base tessellation as the previous figure, although here the valleys between each rhombus is filled in to from a jagged pentagon shape rather than a pinwheel or star shape.

Source: src/5-fold-pentagon-rhombus-filled.json

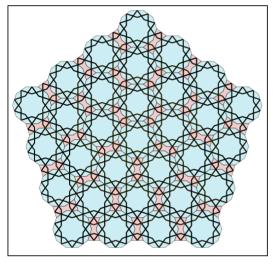


• 5-fold Decagon and Bow-Tie Tessellation

A regular decagon will not tessellate by itself. Adding the bow-tie tiles, allows the decagon to have a fairly simple tessellation. Each sector has in-out and right-left symmetry of the basic 2-fold decagon bow-tie tessellation.

Another way of looking at this figure is that it is composed of super rhombi, with the vertex of each rhombus defined by a decagon.

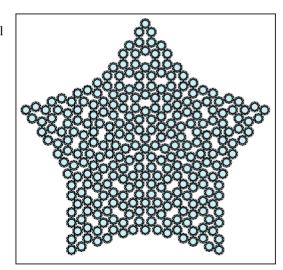
Source: src/5-fold-decagon-bow-tie-tiered.json



• 5-fold Decagon Bow-Tie Hexagon Tessellation with Radial Hexagon Structure

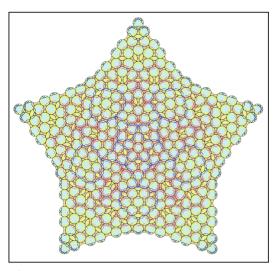
A 5-fold tessellation made with a central decagon, bow-tie and hexagon tiles with a radial hexagon structure. This figure shows the basis skeletal structure. The super hexagon, an elongated but equilateral hexagon, is formed by six touching decagons. Adjacent super hexagons do not overlap, but repeat the overall outline of the shape.

Source: <u>src/5-fold-decagon-bow-tie-hexagon-structure-radial-hexagon-scaffold.json</u>



This shows the preceding skeleton filled in and some of the strapping loops highlighted.

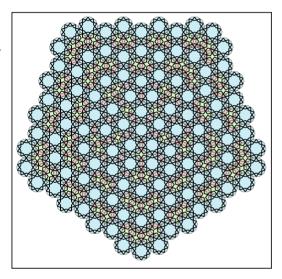
Source: src/5-fold-dbh-radial-hexagon.json



• 5-fold Decagon Bow-Tie Hexagon Tessellation with Tangential Hexagon Structure

A 5-fold tessellation made with a central decagon, bow-tie and hexagon tiles with a tangential hexagon structure. This pattern also has an elongated super hexagon formed by 6 touching decagons. In this pattern, the super hexagons share vertices with adjacent super hexagons.

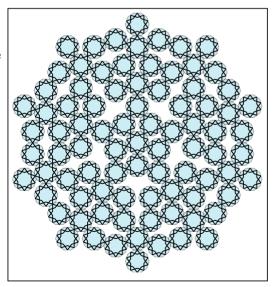
Source: src/5-fold-decagon-bow-tie-hexagon-structure-tangential-hexagon.json



• 5-fold Decagon Bow-Tie Hexagon Tessellation with Pentagon Structure

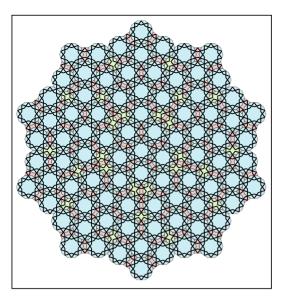
This is a skeleton of a 5-fold tessellation featuring a lot of super pentagons. There is a central super pentagon and the outer ring consists of 10 non-overlapping super pentagons. In between are five more super pentagons that overlap with one of the outer super pentagons. Each of the super pentagons, has a central decagon surrounded by five touchingrdecagons, where the center of each of these is a vertex in the super pentagon.

 ${\color{red} Source: \underline{src/5-fold-decagon-bow-tie-hexagon-structure-pentagon-\underline{scaffold_json}}}$



This figure fills out the preceding skeleton with hexagon and bow-tie tiles.

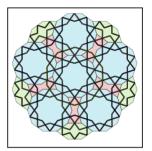
Source: src/5-fold-decagon-bow-tie-hexagon-structure-pentagon.json



• 5-fold Pentagon Made with Decagons and Bow-ties

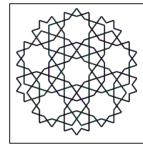
This figure shows a simple pentagon composed of decagons and bow-ties.

Source: simple-penta-deca-bow-tie.json



This figure shows the same pattern, but only showing the strapping. This is the typical way girih is used in architectural settings.

Source: simple-penta-deca-bow-tie.json



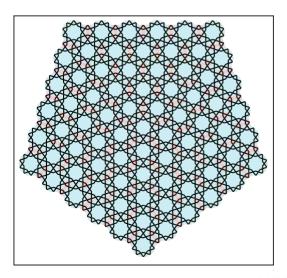
• 5-fold Decagon Bow-Tie

This figure shows the same pattern, but showing the strapping and the underlying tile boundaries. This view lets one appreciate the girih structure.

Source: simple-penta-deca-bow-tie.json

This figure is the same basic pattern as the preceding figure with more tiers.

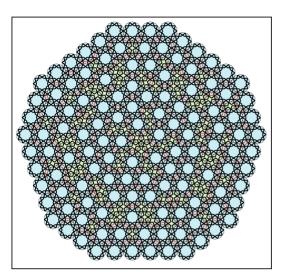
Source: src/penta-deca-bow-tie.json



• 5-fold Hexagon Bow-Tie Decagon

This figure shows a 5-fold tiling with a five central hexagons, bow-ties and decagons.

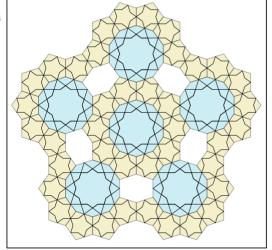
Source: 5-fold-hexagon-bow-tie-decagon.json



• 5-fold Super Decagon and Super Pentagons

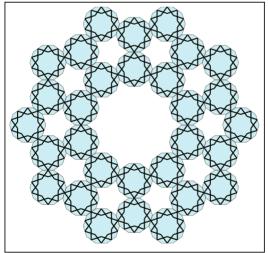
The idea here is to build a tessellation with decagons and pentagons even though neither of them tessellation on their own (or together). As you can see here, there is a hexagonal hole where pentagons should be placed from either end, though they would be overlapping. With girih tiles the hole cannot be filled directly. However if the tiles were super-sized, then maybe girih tiles can fill the hole.

Source: src/5-fold-decagon-pentagon-concept.json



This image shows the core super decagon and super pentagons. The inner ring is a decagon made with a ring of ten decagons. This ring is surrounded by 10 pentagon rings formed by a ring of five decagons. Note that common sides share the pair of points or decagons that define that side.

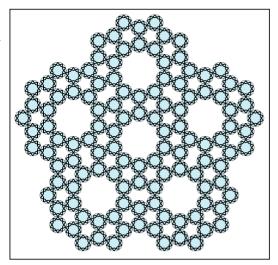
Source: src/super-decagon-pentagon-concept.json



This image shows the recreation of the first structure of this group with super-sized decagon and surrounding pentagons. These super tiles are constructed with rings of decagon tiles to form super decagons and super pentagons. The resulting "hexagonal hole" is now easy to fill with girih tiles as a double star.

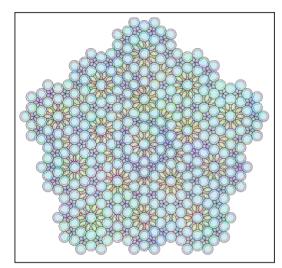
The large holes on the inside of the super decagons can be filled with a decagon, 10 bow-tie and 10 hexagon tiles. The star shaped hole in the center of the super pentagon can be filled with 5 rhombus tiles or a hexagon tile and two bow-ties. The double star hole in the center of the "hexagonal hole" can be filled with three bow-tie and two hexagon tiles.

Source: src/super-decagon-pentagon-skeleton.json



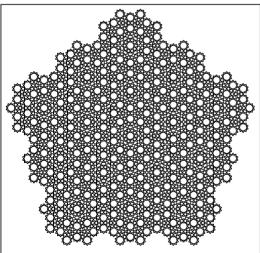
The tessellation is built with 16 super decagons and surrounding super pentagons. The "holes" have been filled mindful of keeping the figure radially symmetric.

Source: src/super-decagon-pentagon-tier.json



The tessellation is the same as the previous, but only the strapping is shown.

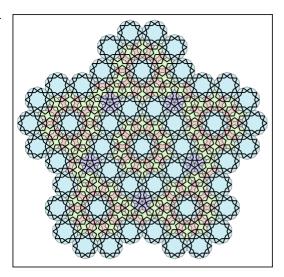
Source: src/super-decagon-pentagon-tier.json



• Decagon Bow-tie Hexagon Rhombus

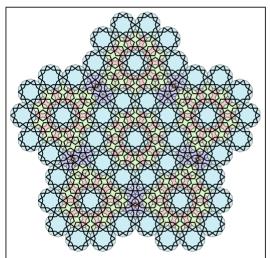
This figure builds a structure of six super-decagons with the outer super-decagons spaced with five super-hexagons.

Source: <u>5-fold-decagon-bow-tie-hexagon-rhombus.json</u>



This figure is an alternative to the previous figure changing the star shape with a double star shape.

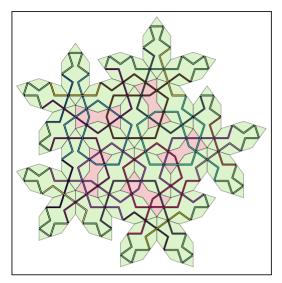
Source: <u>5-fold-dbhlr.json</u>



• 5-fold Hexagon and Bow-Tie Tessellation

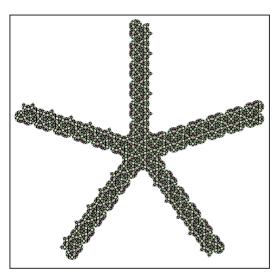
The beginning of a simple 5-fold tessellation using hexagon and bow-tie tiles.

Source: src/5-fold-hexagon-bow-tie-core.json



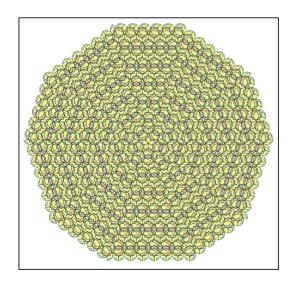
This image shows the spine of an overly ambitious 5-fold tessellation made with hexagon and bow-tie tiles.

Source: src/5-fold-hexagon-bow-tie-spine.json



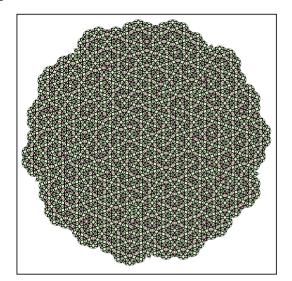
This image shows a 5-fold tessellation made with hexagon and bow-tie tiles. This tessellation has scattered hexagon and pentagon loops.

Source: src/5-fold-hb-tiered.json



This image shows a 5-fold tessellation made with hexagon and bow-tie tiles. This tessellation has many double pentagon hexagon loops. Its internal structure has a repeating pattern.

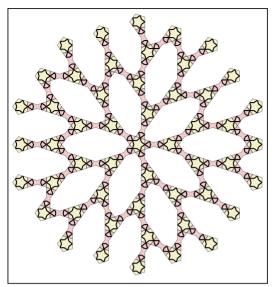
Source: src/5-fold-hb-amorphous/5-fold-hb-amorphous.json



• 5-fold Pentagon, Bow-Tie, Rhombus and Lozenge Tessellation

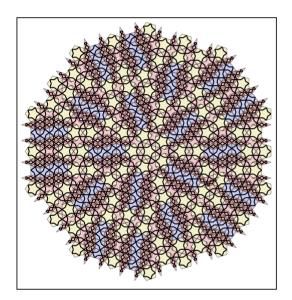
The beginning of a simple 5-fold tessellation using pentagon and bow-tie tiles.

Source: src/5-fold-pentagon-bow-tie-scaffold.json



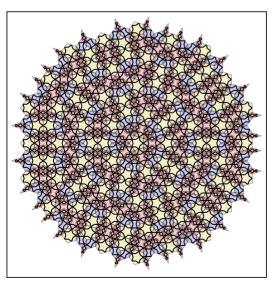
This image shows the skeleton filled in with pentagon, rhombi, and lozenge tiles.

Source: src/5-fold-pblr.json



This image shows the skeleton filled in with an alternate pattern of pentagon, rhombi, and lozenge tiles.

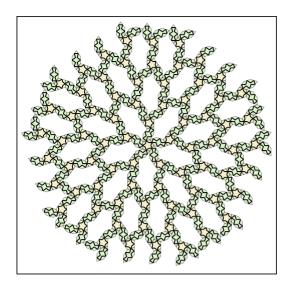
Source: src/5-fold-pblr2.json



• 5-fold Pentagon, Hexagon, Rhombus and Lozenge Tessellation

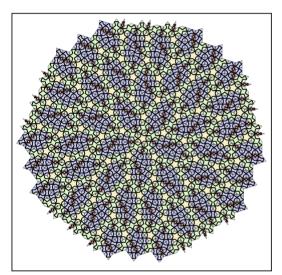
The beginning of a simple 5-fold tessellation using pentagon and hexagon tiles.

Source: src/5-fold-ph-scaffold.json



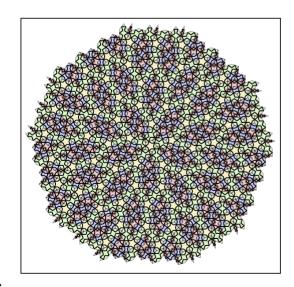
This image shows the skeleton filled in with rhombi and lozenge tiles.

Source: src/5-fold-phlr.json



This image shows the skeleton filled in with an alternate pattern of rhombi, bow-tie, and lozenge tiles.

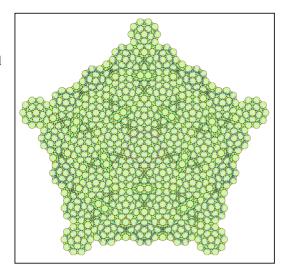
Source: src/5-fold-phlrb.json



• 5-fold Decagon, Bow-tie, Hexagon with a Hexagon Tessellation Structure

This figure started out as central pentagon. The circles enclosing the center were interesting to me, so I wanted to repeat that pattern. This could use either a rhombic pattern or a hexagonal pattern. This version opted for the hexagon form, even though it is quite large to incorporated the circles at each vertex. This has two full layers of the hexagons.

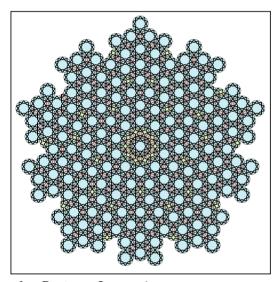
Source: src/5-fold-decagon-bow-tie-hexagon-structure-hexagon-strapping.json



• 5-fold Decagon, Bow-tie, Hexagon with Stars Tessellation Structure

This figure features stars made of 11 decagons. Ten of these overlap to form the center. There are ten more non=overlapping stars around the center.

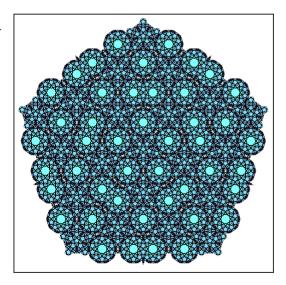
Source: src/5-fold-dbh-stars.json



• 5-fold Pentacular (Pentagon, Lozenge, Bow-tie, Rhombus, Decagon structured as Pentagon Lozenge)

This figure uses a pentagon and lozenge tessellation as a second order fractal. A decagon ringed by 10 pentagons form the vertices of the super pentagon and lozenge. The core or each super pentagon is base-sized version the pentagon-lozenge tessellation.

Source: src/5-fold-plrbd-pl.json



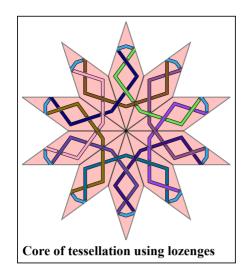
10-fold Tessellations

Girih tiles can make 10-fold tessellations. The basic form is tessellation using the lozenge. Even though this tile is not part of the girih tile set, it sets up the shape symmetrical 10-fold tessellations.

• 10-fold Lozenge Tessellation

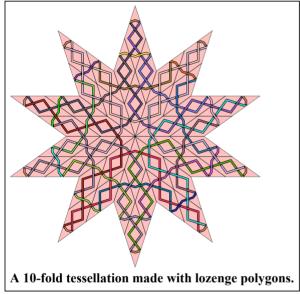
The tessellation is made with 10 lozenges touching a common point.

Source: src/10-fold-lozenge-core.json



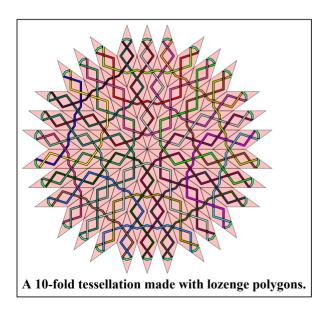
Here the tessellation is expanded out one more tier keeping the star shape.

Source: src/10-fold-lozenge-tiered.json



Alternative the expanded tessellation can fill out the valleys between star points to form a saw toothed decagon.

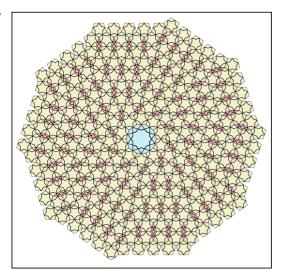
Source: src/10-fold-lozenge-tiered2.json



• 10-fold Decagon Pentagon Lozenge

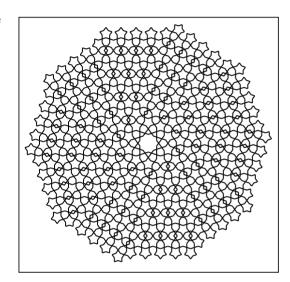
This figure shows a decagon made with a central decagon surrounded by pentagons and lozenges. It is not a traditional girih pattern, but the star strapping of the pentagons shines through.

Source: 10-fold-dpl.json



This figure is the same as the previous pattern with the emphasis on the strapping.

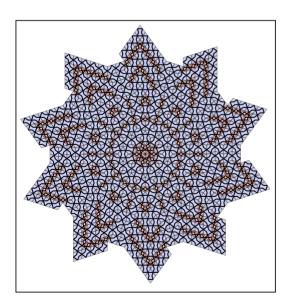
Source: 10-fold-decagon-pentagon-lozenge-strapping.json



• 10-fold Lozenge Rhombus

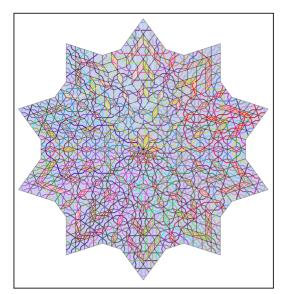
This is a decagon made with a central lozenge rosette surrounded by rhombus and lozenge tiles

Source: <u>10-fold-lozenge-rhombus.json</u>



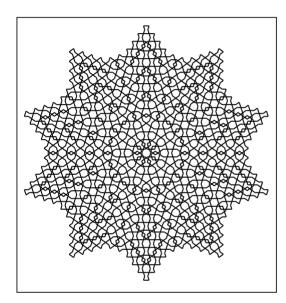
This is a decagon made with a central lozenge rosette surrounded by rhombus and lozenge tiles

Source: 10-fold-lozenge-rhombus1.json



This is the same as the previous figure but emphasizing the strapping.

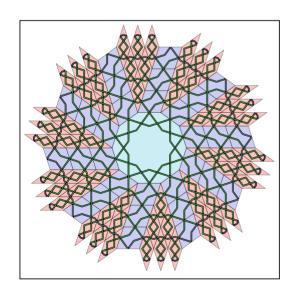
Source: 10-fold-lozenge-rhombus-strapping1.json



• 10 Fold Decagon Rhombus Lozenge

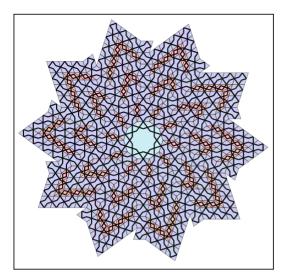
This figure is similar to the previous pattern, although it has less symmetry.

Source: 10-fold-decagon-rhombus-lozenge-simplified1.json



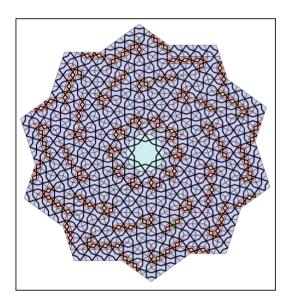
This figure is similar to the previous pattern, although it has less symmetry.

Source: 10-fold-decagon-rhombus-lozenge-simplified2.json



This figure is similar to the previous pattern, although it has less symmetry.

Source: <u>10-fold-decagon-rhombus-lozenge-simplified3.json</u>



This figure is similar to the previous pattern, although it has less symmetry.

Source: <u>10-fold-decagon-rhombus-lozenge-simplified4.json</u>

